

TO: Distribution
FROM: Mark Milman
SUBJECT: Feedforward signal error due to metrology errors

Introduction/Summary. In this memo we consider the effect of metrology errors on the feedforward command for the science star interferometer generated from guide star interferometer internal delay measurements and external metrology measurements of the interferometer baselines. The analysis and results presented here extend some of the work previously reported in [1] and [2].

There are essentially two parts to the process that synthesizes the feedforward command from the metrology measurements. In the first part of the process the position vectors of the guide star and science star interferometer baselines are determined from the one dimensional external metrology measurements. The multiplication factor between metrology measurement error and the resulting baseline 3-D position error is between 9.5 and 10 for the three baselines. In practice these position vectors would be realized with respect to a spacecraft coordinate frame that is subsequently tied to an inertial frame via the on-board attitude determination system. Because the attitude determination system has errors, there is an error between the estimated frame and the inertial frame that describes the star positions. The guide star interferometers are used to make this correction. Both internal metrology and external metrology errors contribute to the final feedforward error, although the external metrology system is shown to be the dominant error source.

After incorporating the external metrology error, the feedforward error is dependent on just a few instrument and star geometry parameters. These parameters are most conveniently defined with respect to the plane, call it P , that is orthogonal to the instrument baseline. These are:

- (1) The “projected” baseline length which is defined as the length of the interferometer baseline times the length of the projection of the star position vector onto P . All quantities involving baseline lengths actually only depend on the projected baseline length.
- (2) The guide and science star position vectors projected onto P . The resulting elevation angle separation between these stars have a significant impact on the noise amplification properties of the internal and external metrology measurement systems as discussed below.

The optimal position of the science star occurs when the projection of the science star onto P is between the projections of the two guide stars onto this plane. Assuming equal, but independent noise statistics for the guide star measurement noise (e.g., if the guide stars are of the same magnitude), the optimal location for the science star is in the middle when the lengths of the projected baselines of the guide star interferometers are the same. In general the optimal location is closer to the guide star whose projected baseline length is greatest.

As the (elevation) separation angle increases between the guide stars, the sensitivity of the error propagation to the position of the science star decreases. A somewhat interesting, although not terribly useful result, is that if the science star is at the optimal location, noise amplification effects are *reduced* when the separation (in elevation) between the guide stars is reduced. But, (and this is a very big but), as the elevation angle between the guide stars becomes small, small deviations in the target star position from optimal can lead to significant degradation in performance.

For example, with a 10° separation between the guide stars, in the best case (where the science star is between the two guide stars) an error amplification factor of approximately 5.5 results. In the worst case, where the science star is offset 30° from one of the guide stars, an amplification factor of 21.5 is obtained. The sensitivity to science star location is reduced with a 30° separation between the guide stars. The worst case now has an amplification factor of 7.3, while the factor

in the best case is 5.9. For a 30° separation there is little sensitivity to the position of the science star. These results are in agreement with Brad Hine's Monte Carlo simulations of star geometries in [1]. Importantly, if only the internal metrology error contribution is considered (and external metrology errors are ignored), the amplification factor in all of these results is reduced by a factor of approximately 5. This result presumably has some relevance to the CLASSIC SIM – SON OF SIM trade.

When the noise variance of the guide star interferometer measurements are not the same (e.g., one star is brighter than the other), the main points of the analysis are still valid by replacing the phrase “projected baseline” in the statements above with “the product of the projected baseline and the reciprocal of the rms noise value”.

As it is clear that the projected positions of the guide stars onto the plane P have substantial impact on the noise characteristics of the feedforward command, it would be wise to choose the baseline orientation to maximize the elevation angle separation. And as a word of caution, if this separation is small, not only are the noise characteristics amplified for science stars not “between” the guide stars, but the error due to nonobservability of roll for the 7 siderostat SIM CLASSIC option comes into play as well [2].

The Analysis. Throughout this analysis we will assume the colinearity of the interferometer baselines.

We begin by addressing how baseline measurements are made using the external metrology system. Let X_1, \dots, X_4 denote the positions of the metrology system corner cubes, and X_5, \dots, X_{11} denote the positions of the siderostat corner cubes in some coordinate system. (See the .m file in the Appendix for the location of these cornercubes.) Distance measurements d_{ij} of the form

$$d_{ij} = |X_i - X_j|, \quad i = 1, \dots, 4; \quad j = 1, \dots, 7 \quad (1)$$

are made. The indexing in (1) above is such that $i < j$. Next define the function $F : R^{67} \rightarrow R^{34}$, with coordinate functions

$$F_{ij}(x, d) = |X_i - X_j| - d_{ij}, \quad (2)$$

where $x \in R^{33}$ defines the 11 corner cube locations in 3-space and $d \in R^{34}$ is the vector of external metrology measurements. Thus solutions to $F(x, d) = 0$ provide the relationship between the positions of each siderostat and the distance measurements.

Intuitively any translation and rotation of the coordinates should also produce a solution. This is reflected in the differential of F . Note that

$$\frac{\partial F_{ij}}{\partial X_i} = \frac{1}{|X_i - X_j|} \langle X_i - X_j, \cdot \rangle. \quad (3)$$

Then given $h, k \in R^3$

$$D_x F \begin{pmatrix} h \\ \vdots \\ h \end{pmatrix} = 0, \quad \text{and} \quad D_x F \begin{pmatrix} k \times X_1 \\ \vdots \\ k \times X_{11} \end{pmatrix} = 0. \quad (4)$$

(Here, “ \times ” denotes the cross product operation.) Hence, the null space of the differential $D_x F$ contains differential translations and rotations, as expected. (In fact the null space is spanned by these vectors.)

The first objective is to determine the differential change in corner cube positions as a function of differential changes in the measurement vector d . To this end we linearize (2) to obtain

$$D_x F \delta x + D_d F \delta d = 0. \quad (5)$$

Now since $D_x F$ has a six dimensional null space, we introduce the orthogonal matrix $V : R^{27} \rightarrow R^{33}$ with range orthogonal to this null space to normalize least squares solutions to (5) above. Thus we replace (5) with

$$D_x F V \delta u + D_d F \delta d = 0, \quad (6)$$

determine δu via

$$\delta u = -(V^T D_x F^T D_x F V)^{-1} V^T D_x F^T \delta d, \quad (7)$$

and then obtain δx as $\delta x = V u$. (For variance calculations this normalization leads to a smaller error than if unique solutions were enforced by constraining coordinates. Furthermore, instead of using (7), the pseudoinverse solution could also have been used directly from (5).)

Now define the baseline vectors b_1 and b_2 corresponding to the two guide interferometer baselines as $b_1 = X_6 - X_5$, $b_2 = X_8 - X_7$, and define the science interferometer baseline as $b_3 = X_{10} - X_9$. The relationship between the baselines and the siderostat positions is given by solutions to

$$R(x, b) = 0, \quad \text{where} \quad R(x, b) = Rx - b, \quad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \quad (8)$$

and R is the matrix that produces the vector b from the siderostat positions as defined above.

Now δb is obtained from (7) and (8) as

$$\delta b = V(D_x F)^\dagger \delta d. \quad (9)$$

Note that the variance of the baseline measurements $Q = E(\delta b \delta b^T)$ is given as

$$Q = R V [V^T D_x F^T D_x F V]^{-1} V^T R^T. \quad (10)$$

For this configuration

$$\sqrt{\text{diag}(Q)} = (7.259, 5.463, 3.983, 7.042, 5.811, 3.592, 6.866, 5.833, 3.295),$$

where the rms errors of the (x, y, z) coordinates of b_1 are contained in the first triple of numbers, the errors of b_2 in the next three, etc. Somewhat unintuitively we find that the variance of the x coordinate for each baseline is always the largest. This is due to the fact that the metrology tetrahedron is not fixed. (A sanity check was performed by fixing X_1, \dots, X_4 . In this case the line of sight from the beam launchers to the siderostats – largely in the x direction – had the smallest variance, as expected.)

Next we will focus on how internal metrology measurements together with the baseline measurements produce the feedforward term.

Let s_1 and s_2 denote the unit vectors giving the directions for guide stars 1 and 2, and let s_3 denote the unit vector giving the direction of the science star. Let y_1 and y_2 represent the measured pathlength delays for the guide star interferometers, and let z represent the pathlength delay for the science star. The delay z is synthesized from the delay measurements y_1 , y_2 , and from the external metrology measurements of the vectors b_1 , b_2 , and b_3 . We note that

$$y_1 = \langle s_1, b_1 \rangle, \quad (11)$$

$$y_2 = \langle s_2, b_2 \rangle, \quad (12)$$

where the inner product is in any coordinate system. The feedforward signal for the science interferometer is then

$$z = \langle s_3, b_3 \rangle. \quad (13)$$

The stumbling block here is that the metrology measurements of the baseline vectors are made with respect to the metrology coordinate system, and the transformation between this and the inertial coordinate system is initially known only through standard attitude determination knowledge, which is not of sufficient precision to synthesize z to the required tolerances. Thus in general the star positions and the baseline vectors are known in different coordinate systems, and z cannot be directly synthesized. We let U denote the rotation between the ACS coordinate system and the inertial frame. Because this error is on the order of a few arcsec, we assume that U is close to the identity matrix. Hence,

$$z = \langle s_3, U b_3 \rangle, \quad (14)$$

where the coordinates of b_3 are in the ACS frame and the coordinates of s_3 are in the inertial frame. We write this equation as

$$H(z, U, b) = 0. \quad (15)$$

Similarly we write

$$y_1 = \langle s_1, U b_1 \rangle, \quad (16)$$

$$y_2 = \langle s_2, U b_2 \rangle, \quad (17)$$

with the coordinates of b_1 and b_2 in the ACS frame and the coordinates of s_1 and s_2 in the inertial frame. We write these equations as

$$G(y, U, b) = 0. \quad (18)$$

To assess the effect of metrology measurement error on the feedforward command, the differential of z as a function of the differentials of the external metrology measurement d and the internal metrology measurement y are needed. This requires in addition to dF , the differentials of H and G in (15) and (18) with respect to the collection of independent variables x, d, b, y, U, z . The corresponding differentials are denoted $\delta x, \delta d, \delta b, \delta y, \delta U, \delta z$. We note that since U is nominally the identity matrix, δU can be identified with the 3-vector ω via the cross product correspondence: $\delta U x = \omega \times x$ for any vector x .

Thus we have:

$$dH = \langle s_3, \omega \times b_3 \rangle + \langle s_3, \delta b_3 \rangle - \delta z, \quad (19)$$

$$dG = T\omega + S\delta b - \delta y, \quad (20)$$

where

$$T = \begin{pmatrix} s_1 \times b_1 \\ s_2 \times b_2 \end{pmatrix}, \quad S\delta b = \begin{pmatrix} \langle s_1, \delta b_1 \rangle \\ \langle s_2, \delta b_2 \rangle \end{pmatrix}, \quad \text{and} \quad \delta y = \begin{pmatrix} \delta y_1 \\ \delta y_2 \end{pmatrix}. \quad (21)$$

$$dR = R\delta x - \delta b, \quad (21)$$

$$dF = D_x F \delta x + D_b F \delta b. \quad (22)$$

We can solve for ω from (20) above:

$$\omega = T^\dagger (\delta y - S\delta b). \quad (23)$$

The analysis in [2] shows that the component of ω in $N(T)$ makes a negligible contribution to δz in (19) when the baselines are colinear.

These computations result in the linearization

$$\delta z = \langle s_3, T^\dagger(\delta y - SRD_x F^\dagger \delta d) \times b_3 \rangle + \langle s_3, \Pi_3 RD_x F^\dagger \delta d \rangle, \quad (24)$$

where Π_3 projects a 9-vector onto its last three coordinates. Then treating δy and δd as independent random vectors (with scalar covariance matrices $\sigma I_{2 \times 2}$ and $\sigma I_{34 \times 34}$, respectively), we obtain

$$E(|\delta z|^2) = \sigma^2 |T^{\dagger T}(s_3 \times b_3)|^2 + (S^T T^{\dagger T}(s_3 \times b_3) + \Pi_3 s_3)^T Q (S^T T^{\dagger T}(s_3 \times b_3) + \Pi_3 s_3). \quad (25)$$

Although this is at first glance a somewhat unwieldy expression, the influence of the relative positions of the guide stars and science star on the variance can be analyzed. We take this up next.

First define the 2×2 covariance matrix

$$Q_2 = S Q S^T. \quad (26)$$

Using (24) we have the (fairly accurate) bound

$$\sqrt{E(|\delta z|^2)} \leq \sigma |(Q_2 + I)^{1/2} T^{\dagger T}(s_3 \times b_3)| + \sigma |s_3^T E(\delta b_3 \delta b_3^T) s_3|^{1/2}. \quad (27)$$

The location of s_3 within the instrument's $30^\circ \times 30^\circ$ FOV has little effect on the second term above. We will investigate its affect on the first term.

Note the inequality

$$\sigma_{min}[(Q_2 + I)^{1/2} T^{\dagger T}] \leq \frac{1}{|s_3 \times b_3|} |(Q_2 + I)^{1/2} T^{\dagger T}(s_3 \times b_3)| \leq \sigma_{max}[(Q_2 + I)^{1/2} T^{\dagger T}], \quad (28)$$

where σ_{min} , σ_{max} denote the smallest and largest singular values, respectively. In fact we have the decomposition

$$|(Q_2 + I)^{1/2} T^{\dagger T}(b_3 \times s_3)|^2 = \sigma_{min}^2 < b_3 \times s_3, v_1 >^2 + \sigma_{max}^2 < b_3 \times s_3, v_2 >^2, \quad (29)$$

where v_1 and v_2 denote the associated singular vectors of $(Q_2 + I)^{1/2} T^{\dagger T}$.

Hence, by (28) the error can be bounded by the singular values. And (29) shows that these bounds can actually be achieved by appropriately chosen science star positions. (We will show later that the minimum value occurs within the instrument's field of view.) The figures below show the minimum and maximum singular values for $(Q_2 + I)^{1/2} T^{\dagger T}$ as the elevation angle difference between the two guide stars is increased from one to thirty degrees.

It is interesting to note that σ_{min} is insensitive to the separation angle and actually increases as the separation between the guide stars increases, while σ_{max} decreases. The important message here is that if there is relatively large elevation angle separation between the guide stars, there is a small difference between the maximum and minimum singular values of $(Q_2 + I)^{1/2}T^{\dagger T}$; and consequently the variance of the feedforward term changes slowly with changes in the elevation angle of the science star. (Consequently, there is relative insensitivity to the position of the science star within the field of view.) This will not be the case if the guide stars are close together in elevation.

The location of the singular vectors determines the science star directions that are most (least) sensitive. It is fortunate that it turns out that the singular vectors of $(Q_2 + I)^{1/2}T^{\dagger T}$ are closely approximated by the singular vectors of $T^{\dagger T}$. For completeness we will reprise some of the analysis in [2] that characterizes these singular vectors.

Let the singular value decomposition of $T^{\dagger T}$ be given as

$$T^{\dagger T} = U \Sigma V^T. \quad (30)$$

Let $\sigma_1 \geq \sigma_2$ denote the nonzero singular values of $T^{\dagger T}$, and let v_1 and v_2 denote the first two rows of V^T . Thus s^* is the optimal direction when $b_3 \times s^*$ is in the subspace spanned by v_2 .

Now the columns of V are the eigenvectors of $T^T T$, and the singular values are given as $\sigma_1 = 1/\sqrt{\nu_-}$ and $\sigma_2 = 1/\sqrt{\nu_+}$ where ν_{\pm} are the nonzero eigenvalues of $T^T T$. Note that

$$T^T T = (s_1 \times b_1)(s_1 \times b_1)^T + (s_2 \times b_2)(s_2 \times b_2)^T,$$

and

$$(s_i \times b_i)(s_i \times b_i)^T = b_i^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & s_i^2(3) & -s_i(2)s_i(3) \\ 0 & -s_i(2)s_i(3) & s_i^2(2) \end{pmatrix}$$

From this representation the eigenvalues of $T^T T$ are easily computed as

$$\nu_{\pm} = \frac{1}{2} \{ (\alpha + \gamma) \pm \sqrt{(\alpha - \gamma)^2 + 4\beta^2} \},$$

where α, β, γ are the (2,2), (2,3) and (3,3) components of $T^T T$. Specifically

$$\begin{aligned}\alpha &= b_1^2 s_1^2(3) + b_2^2 s_2^2(3), \\ \beta &= -b_1^2 s_1(2)s_1(3) - b_2^2 s_2(2)s_2(3), \\ \gamma &= b_1^2 s_1^2(2) + b_2^2 s_2^2(2).\end{aligned}$$

The associated non-normalized eigenvectors to ν_{\pm} are then

$$\omega_+ = \begin{pmatrix} 0 \\ \beta \\ \nu_+ - \alpha \end{pmatrix}, \quad \omega_- = \begin{pmatrix} 0 \\ \beta \\ \nu_- - \alpha \end{pmatrix}$$

ω_+ and ω_- will now be used to characterize the best and worst positions for the science star relative to the guide stars. The first thing to note is that these vectors are orthogonal, hence, the best and worst directions are orthogonal to each other. This orthogonality also provides a canonical decomposition of the science star vector to characterize its associated noise amplification properties.

The characteristic polynomial for $T^T T$ is $p(s) = (s - \alpha)(s - \gamma) - \beta^2$. Thus, $p(\alpha) < 0$, and it follows since p is quadratic with $p(\nu_{\pm}) = 0$ that

$$\nu_- \leq \alpha \leq \nu_+.$$

These inequalities already provide information regarding the quadrants in which ω_+ and ω_- lie. If $\beta > 0$, then $\pm\omega_+$ is in the first and third quadrants of the $e_2 - e_3$ plane, and if $\beta < 0$, $\pm\omega_+$ is in the second and fourth quadrants. We'll assume that the science and guide stars are in the upper half plane, that is $s_1(3), s_2(3), s^*(3) > 0$. In fact we may assume without loss of generality that $s_1(3) = 1$, and s_1 corresponds to the guide star with the longer projected baseline onto the $e_2 - e_3$ plane. This involves a change of coordinates, and we will take a small digression here to formalise this assertion.

Let U denote the rotation matrix in the $e_2 - e_3$ plane such that $Us_1 = e_3$. Note here that $Ue_1 = e_1$. We claim that $\omega_{\pm} \rightarrow U\omega_{\pm}$ (and consequently the same transformation applies to ω_{opt}). To see this, note that for any vector w ,

$$\begin{aligned}TU^T w &= \begin{pmatrix} \langle s_1 \times b_1, U^T w \rangle \\ \langle s_2 \times b_2, U^T w \rangle \end{pmatrix} \\ &= \begin{pmatrix} \langle Us_1 \times b_1, w \rangle \\ \langle Us_2 \times b_2, w \rangle \end{pmatrix}\end{aligned}$$

since $Ub_i = b_i$. Thus if $T^T T\omega_{\pm} = \nu_{\pm}\omega_{\pm}$, it follows that $U\omega_{\pm}$ is an eigenvector of $UT^T TU^T$. Noting the form of TU^T , we see this is just the rotation of the guide star vectors s_1 and s_2 .

Since $b_3 \times s^* = [0 \quad -s^*(3) \quad s^*(2)]$, it follows that the optimal (un-normalized) direction for s^* is

$$\omega_{opt} = [0 \quad \alpha - \nu_+ \quad \beta], \quad \text{if } \beta > 0,$$

and

$$\omega_{opt} = [0 \quad \nu_+ - \alpha \quad -\beta], \quad \text{if } \beta < 0.$$

If $\beta = 0$ then the direction is e_3 .

Note that since b_1 is the long baseline, $\alpha > \gamma$. (This inequality also holds if b_1 is the short baseline with the additional requirement that s_2 is within $\pi/4$ radians of s_1 . This observation will be important shortly.) Therefore

$$\begin{aligned}\nu_+ - \alpha &= \frac{1}{2}(\gamma - \alpha) + \frac{\sqrt{(\gamma - \alpha)^2 + 4\beta^2}}{2} \\ &\leq \beta\end{aligned}$$

Hence, the angle between ω_{opt} and e_3 is always less than $\pi/4$ rad. We can use this to show that ω_{opt} is actually between s_1 and s_2 . First note that if the angle between s_2 and e_3 is more than $\pi/4$ rad, then the statement is true. If not, assume to the contrary that s_2 is between e_3 and ω_{opt} . Now introduce the rotation matrix R such that $Re_1 = e_1$ and $Re_2 = e_3$. The result of this rotation is that Rs_1 and $R\omega_{opt}$ are not in the same quadrant. Now, we have shown above that $R\omega_{opt}$ is the optimal direction for guide stars Rs_1 and Rs_2 . And since the angle between Rs_1 and Rs_2 is less than $\pi/4$ rad, we must have Rs_1 and $R\omega_{opt}$ in the same quadrant. This is a contradiction, so ω_{opt} must be between s_1 and s_2 .

We can go a little bit further with this analysis to show that ω_{opt} is actually closer to the guide star with the longer projected baseline. The idea is straightforward. We compare the rotation θ' required to bring ω_{opt} to e_3 with half the angle θ required to bring s_2 to e_3 . We will show that $\theta' \leq \theta/2$ when s_1 has the longer projected baseline, and $\theta' \geq \theta/2$ when s_2 has the longer projected baseline.

First note that ω_{opt} is aligned with e_3 when $\beta = 0$. We will compute the rotation in the $e_2 - e_3$ plane required to produce this. Let

$$R(\theta') = \begin{pmatrix} \cos(\theta') & -\sin(\theta') \\ \sin(\theta') & \cos(\theta') \end{pmatrix}$$

We need to solve

$$b_1^2 s_1^{\theta'}(2) s_1^{\theta'}(3) + b_2^2 s_2^{\theta'}(2) s_2^{\theta'}(3) = 0,$$

where

$$s_i^{\theta'} = R(\theta') s_i.$$

After a little bit of algebra, we arrive at

$$\tan(2\theta') = \frac{2b_2^2 s_2(2) s_2(3)}{b_1^2 + b_2^2 (s_2^2(3) - s_2^2(2))}$$

Now we'll compare this with the rotation θ required to send s_2 to e_3 . Note that $\cos(\theta) = s_2(3)$, and $\sin(\theta) = s_2(2)$, so that

$$\tan(\theta) = \frac{s_2(2)}{s_2(3)}.$$

Thus $\tan(2\theta') \leq \tan(\theta)$ if and only if

$$\frac{2b_2^2 s_2(2) s_2(3)}{b_1^2 + b_2^2 (s_2^2(3) - s_2^2(2))} \leq \frac{s_2(2)}{s_2(3)}.$$

Cross multiplying gives

$$2b_2^2 s_2(3)^2 \leq b_1^2 + b_2^2 (s_2(3)^2 - s_2(2)^2),$$

or

$$2s_2(3)^2 \leq \frac{b_1^2}{b_2^2} + (s_2(3)^2 - s_2(2)^2),$$

and

$$s_2(2)^2 + s_2(3)^2 \leq \frac{b_1^2}{b_2^2}.$$

But $s_2(2)^2 + s_2(3)^2 = 1$, hence, $\tan(2\theta') \leq \tan(\theta)$ if and only if $b_1 \geq b_2$. Since the tangent is an increasing function of θ , ω_{opt} is closer to s_1 than s_2 .

Numerical Results. Based on the preceding analysis, the propagation of the feedforward error is dependent on just a few instrument parameters. These include

- (1) The length of the interferometer baselines times the length of the projection of the star position vector onto the plane orthogonal to the instrument baseline.
- (2) The guide and science star position vectors projected onto the plane orthogonal to the instrument baseline.

With these considerations we assumed baseline lengths of 7m and 8m for the guide star interferometers, and a length of 10m for the science interferometer baseline. We always assumed the guide stars and science stars to be in the plane orthogonal to the instrument baseline. (If not, project the star onto the plane and multiply the associated baseline with the length of the projected star position vector.)

Using a $30^\circ \times 30^\circ$ FOV, we considered guide star positions separated by 10° , 20° , and 30° in this plane. The best and worst positions for the science star approximately correspond to a position in the middle of the two guide stars, and maximally separated within the FOV, respectively. The table below contains these results. Columns 2 and 3 include both external and internal metrology errors, while Columns 4 and 5 include only internal metrology errors. The results contained in these last two columns are probably representative of the Son of SIM error propagation.

RMS Multiplication Factor as Function of Star Geometry

Separation Angle	Worst Case*	Best Case*	Worst Case**	Best Case**
10°	21.464	5.4861	4.568	.9527
15°	13.515	5.5423	2.806	.9573
20°	9.9547	5.629	1.966	.9637
30°	7.3188	5.887	1.250	.9826

* Propagation factor includes external and internal metrology errors

** Propagation factor excludes external metrology error

Although these numbers were calculated from the formula (25), they followed the bounds established in (27) rather closely. Also, the analytical predictions based on the singular value analysis are reflected in this data. For example, the locations of the science star yielding the maximum and minimum sensitivities, the trend of the sensitivities as the guide star elevation angle separation is increased, and finally the slight increase in the variance of the minimum sensitivity location as the separation is increased, are all corroborated by the data. With regards to the SIM CLASSIC vs. SON of SIM trades, note that when the contribution of the external metrology error is ignored, a factor of about 5 improvement is realized for all the cases.

References

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- [2] M. Milman, Feedforward signal and guide star interferometer geometry, IOM 3456-96-029, June, 1996.

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